TONBRIDGE SCHOOL

Scholarship Examination 2017

MATHEMATICS I

Monday 24th April 2017
11.30 am

Time allowed: 1 hour 30 minutes

Answer as many questions as you can.
Questions 1 to 5 are worth 8 marks each;
Questions 6 to 9 are worth 15 marks each.

All answers must be supported by adequate explanation.
Calculators may be used in any question.

1. Solve the simultaneous equations:

$$\tfrac{1}{2}x+\tfrac{1}{3}y=\tfrac{1}{4}$$
$$2x+3y=4$$

[8 marks]

2. Alan and Brian are two runners. Alan runs at 6.6 m/s and Brian runs at 8.4 m/s. They start from the same place on a circular 400 m track and run in opposite directions round and round the track.

(a) How far has Brian run when they pass each other for the first time?

(b) After they pass each other for the second time, how much further does Brian need to run in order to complete two laps (800 m)?

[8]

3. The total surface area, T, of a solid hemisphere is connected to its volume, V, by the formula $4T^3 = 243\pi V^2$.

(a) If $V = 240\,\text{cm}^3$, find T.

(b) In an experiment, T is measured as 30 cm^2 and V as 12 cm^3.
What value of π does the formula then give?

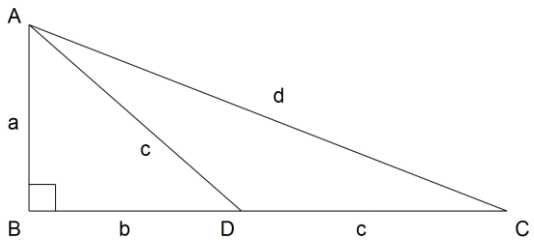
(c) If the values of T and V are numerically the same, find T.

[8]

4. In the right-angled triangle below, $AD = DC$ and a, b, c, d denote the lengths in cm of the sides shown.

(a) If $a = 24,\ b = 7$ find c and d.

(b) If $a = 120,\ c = 169$ find b and d.



[8]

5. A piece of string 231 cm long is to be cut into two pieces. Find the lengths of the two pieces in each of the following cases.

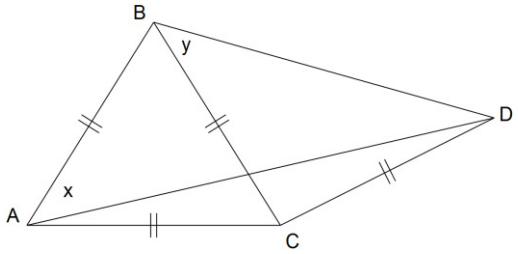
(a) One piece is 20 times the length of the other.

(b) One piece is 20 cm longer than the other.

(c) One piece is 20% longer than the other.

[8]

6. In the figure below, the marked lengths AB, AC, BC, CD are all equal.



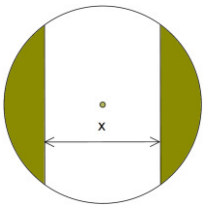
(a) If $x = 23°$, find angle y.

(b) Use algebraic reasoning to find y in terms of x, simplifying your answer as far as possible.

(c) If triangle BCD is equilateral, what is the value of x?

[15]

7. The figure below shows the cross-section of a sphere of radius 3 cm with the two identical shaded regions, distance x cm apart, chopped off. The volume, y cm^3, of the shape that remains is given by the formula $y = \dfrac{\pi x(108 - x^2)}{12}$.



(a) When $x = 3$, show that $y = 77.8$ (correct to 1 decimal place).

(b) Find the values of y for $x = 0, 1, 2, 4, 5, 6$.

(c) Choosing sensible scales, use your values in (a) and (b) to plot a graph of y against x.

(d) What is the value of x when $y = 85$?

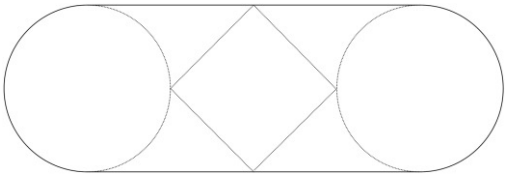
(e) Use your answer to either (b) or (c) to find the volume of the whole sphere.

[15]

TURN OVER

8. The inside of the figure below is made up of a square of side 8 cm which touches two identical circles.

(a) Find the diameter of the circles.



The rest of this question concerns the region enclosed by the outer solid line round the edge of the diagram.

(b) Find the length of the perimeter of this region.

(c) Find the area of this region.

[15]

9. The sequence of *triangle numbers* is given by $T_1 = \frac{1\times 2}{2} = 1,\ T_2 = \frac{2\times 3}{2} = 3,\ T_3 = \frac{3\times 4}{2} = 6,$ etc..

In the table below, Column B is the answer to the sum shown in Column A.

	A	**B**
Row 1	T_1	
Row 2	$T_1 - T_2 + T_3$	4
Row 3	$T_1 - T_2 + T_3 - T_4 + T_5$	
Row 4		
Row *n*		

(a) What are the entries in Column B for Rows 1, 3 and 4?

(b) Find formulae in terms of n for:
(i) The entry in Column B for Row n;
(ii) The number of triangle numbers in Column A for Row n.

(c) If the entry in Column A ends with T_{97}, what is the corresponding entry in Column B?

(d) If the right-hand triangle number in Column A is 7626, what is the Row number?

[15]

END OF PAPER